Precalc Warm Up # 9-1

If the following is an identity, prove it. If it is not an identity find all solutions in the interval $[0,2\pi)$

1. $\cos x - \dfrac{\cos x}{1-\tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$

2. $2\cos x - 1 = 0$

HW Questions:

15) $\underbrace{\sin^{1/2}x\cos x} - \underbrace{\sin^{5/2}x\cos x} = \cos^3 x\sqrt{\sin x}$

$\sin^{1/2}x$

$\sin^{1/2}x\cos x - \sin^{1/2}x\sin^{4/2}x\cos x$

think: $x^{5/2}$
$= x^{\frac{1}{2}+\frac{4}{2}}$
$= x^{1/2}\cdot x^{4/2}$
$(\sin x)^{1/2}(\sin x)^{4/2}$

factor out the greatest common factor.

$\sin^{1/2}x\cos x(1-\sin^2 x)$

$\sqrt{\sin x}\cos x(\cos^2 x)$

$\cos^3 x\sqrt{\sin x} = \cos^3 x\sqrt{\sin x}$

27) $\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1}$

* give them common denominator then carefully multiply out the denominator

31) $\underbrace{2 + \cos^2 x - 3\cos^4 x}_{}$ $= \sin^2 x\,(2 + 3\cos^2 x)$

factor this quadratic.
Take a hint from this factor

35) factor difference of squares!

39) $\dfrac{\tan^3\alpha - 1}{\tan\alpha - 1}$ $= \tan^2\alpha + \tan\alpha + 1$

use: $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$$\frac{\cancel{(\tan\alpha - 1)}(\tan^2\alpha + \tan\alpha + 1)}{\cancel{\tan\alpha - 1}}$$

$\tan^2\alpha + \tan\alpha + 1 = \tan^2\alpha + \tan\alpha + 1$

47) Create some $\frac{\sin x}{\cos x}$'s to make $\tan x$

$$\frac{\frac{1}{\cos x \cos y}\cdot(\sin x \cos y + \cos x \sin y)}{\frac{1}{\cos x \cos y}\cdot(\cos x \cos y - \sin x \sin y)} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\frac{\frac{\sin x}{\cos x}\frac{\cancel{\cos y}}{\cancel{\cos y}} + \frac{\cancel{\cos x}}{\cancel{\cos x}}\frac{\sin y}{\cos y}}{1\frac{\cancel{\cos x}\cancel{\cos y}}{\cancel{\cos x}\cancel{\cos y}} - \frac{\sin x}{\cos x}\frac{\sin y}{\cos y}} =$$

$$\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

51) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \cdot \sqrt{\frac{1+\sin\theta}{1+\sin\theta}} = \frac{1+\sin\theta}{|\cos\theta|}$

$$\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} =$$

$$\sqrt{x^2} = (\sqrt{x})^2$$

$$\sqrt{x^2} = |x|$$

☆ $1+\sin\theta$ { will always be positive or zero; 1 + something btwn -1 & 1 }

but $\cos\theta$ could be $-$, so we need absolute value when we $\sqrt{}$

55) $-\ln(1+\cos\theta) = \ln(1-\cos\theta) - 2\ln|\sin\theta|$

$$\ln(1-\cos\theta) - \ln(\sin\theta)^2$$

$$\ln\left(\frac{1-\cos\theta}{\sin^2\theta}\right)$$

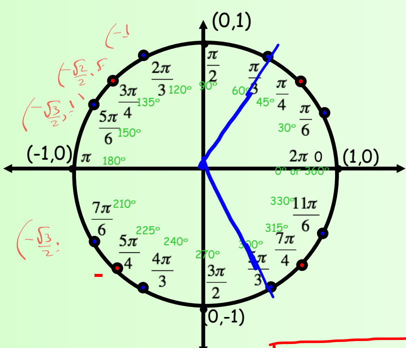
$$\ln\left(\frac{1-\cos\theta}{1-\cos^2\theta}\right)$$

$$\ln\left(\frac{1-\cancel{\cos\theta}}{(1-\cancel{\cos\theta})(1+\cos\theta)}\right)$$

$$\ln\left(\frac{1}{1+\cos\theta}\right)$$

$$\ln(1+\cos\theta)^{-1}$$

Find the GENERAL SOLUTION to: $2\cos x - 1 = 0$



$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

on $[0, 2\pi) \rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

Let $n =$ Integer

$$\boxed{\text{General Solution: } x = \frac{\pi}{3} + 2\pi n \text{ and } x = \frac{5\pi}{3} + 2\pi n}$$

All of the equation solving techniques that you have used in the past to solve equations work on trig equations as well, including

1. simplifying equation by adding like terms together
2. square rooting both sides
3. If you have a quadratic, you might need to factor.
4. If you have a quadratic that can't be factored, you may need the quadratic formula.
5. You may need to square both sides of an equation in order to take advantage of pythagorean identities. You will have to check your solution as you may have created an extraneous solution.
6. Solving equations that have multiple angles, such as cos 2x = 1 can be particularly tricky, especially when trying to find all the solutions from $[0,2\pi)$

Find all solutions from $[0,2\pi)$, then find the general solution.

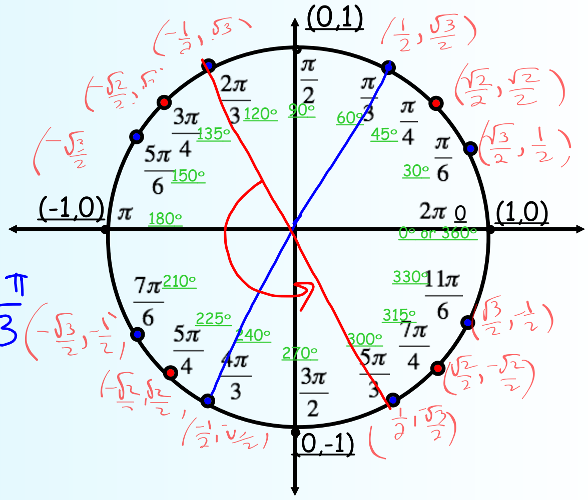
1. $\sqrt{\tan^2 x} = \pm\sqrt{3}$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} + \pi n$

$x = \frac{2\pi}{3} + \pi n$



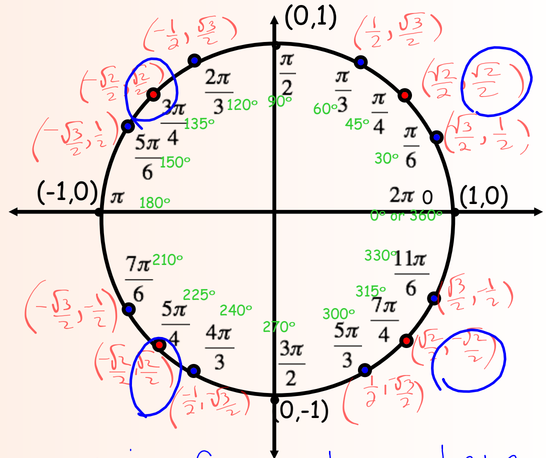
2. $\csc^2 x - 2 = 0$

$\csc^2 x = 2$

$\csc x = \pm\sqrt{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$



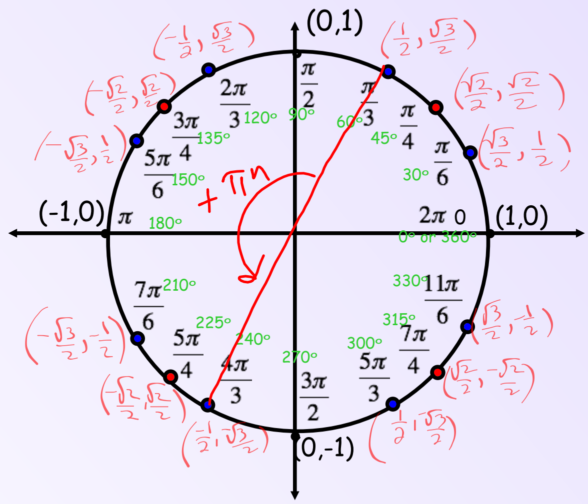
looking for angles where y coordinate on unit circle is $\pm \frac{\sqrt{2}}{2}$

on $[0, 2\pi)$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

General Solution $\longrightarrow$ $x = \frac{\pi}{4} + \frac{\pi}{2}n$

3. $\sin^2 x = 3\cos^2 x$



$1 - \cos^2 x = 3\cos^2 x$

$1 = 4\cos^2 x$

$\frac{1}{4} = \cos^2 x$

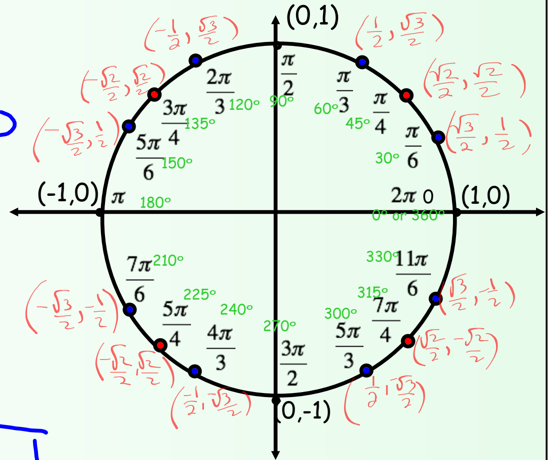
$\pm\frac{1}{2} = \cos x$

$x = \cos^{-1}\left(\pm\frac{1}{2}\right)$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} + \pi n \;,\; \frac{2\pi}{3} + \pi n$

4. $\sin^2 x - \sin x - 2 = 0$



$(\sin x - 2)(\sin x + 1) = 0$

$\sin x = 2$

$\sin x = -1$

$x = \frac{3\pi}{2}$

$$\boxed{x = \frac{3\pi}{2} + 2\pi n}$$

Find all solutions $[0, 2\pi)$ to the nearest hundredth . You will need a calculator (radian mode).

5. $12\cos^2 x + 5\cos x - 3 = 0$

$(4\cos x + 3)(3\cos x - 1) = 0$

$\frac{4\cos x}{4} = -\frac{3}{4}$

$\cos x = -\frac{3}{4}$

$x = \cos^{-1}\left(-\frac{3}{4}\right)$

$\boxed{x \approx 2.42}$

$x = \pi + \theta'$

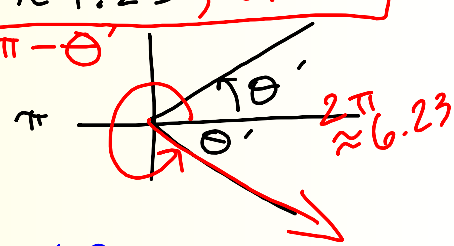
$\boxed{x \approx 3.86}$

$\frac{3\cos x}{3} = \frac{1}{3}$

$x = \cos^{-1}\left(\frac{1}{3}\right)$

$\boxed{x \approx 1.23,\ 5.56}$

$x = 2\pi - \theta'$



$0 \leq \theta \leq \pi$

Multiple angle problems: Even when you are asked to find all solutions on the interval $[0,2\pi)$ you should first find the general solution, then use it to generate all of the solutions on the given interval.

6. sec 4x = 2

$$\cos 4x = \frac{1}{2}$$

$$4x = \cos^{-1}\left(\frac{1}{2}\right)$$

$4x$ on $[0, 2\pi)$:

$$4x = \frac{\pi}{3}, \frac{5\pi}{3}$$

General Solution

$$\frac{4x}{4} = \frac{1}{4}\cdot\frac{\pi}{3} + \frac{2\pi n}{4}$$

$$\boxed{x = \frac{\pi}{12} + \frac{\pi}{2}n}$$

$$\frac{4x}{4} = \frac{5\pi}{3}\cdot\frac{1}{4} + \frac{2\pi n}{4}$$

$$\boxed{x = \frac{5\pi}{12} + \frac{\pi}{2}n}$$

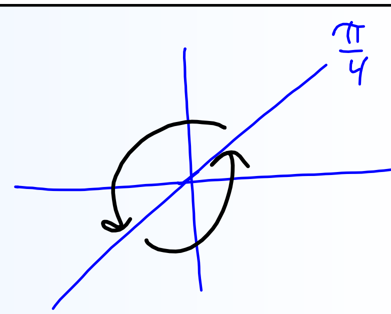
★ Now use the general solutions to list x on $[0, 2\pi)$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

8. 1 - tan 3x = 0

$$1 = \tan 3x$$

$$3x = \tan^{-1}(1)$$



$$\frac{3x}{3} = \left(\frac{\pi}{4} + \pi n\right)\frac{1}{3}$$

$$\frac{5\pi}{4}$$

$$\frac{24\pi}{12}$$

$$\boxed{x = \frac{\pi}{12} + \frac{\pi}{3}n}$$

on $[0, 2\pi)$

$$\frac{4\pi}{12}$$

$$\frac{3\pi}{4}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$$

HW: PC book p. 422

#7-39 every other odd,

and # 41 - 51 odd

HW week 8: pages 403, 411

(2 days for each page)